Solution by H. B. LEONARD, B. S.

From $ax^2+by^2+cz^2=0=\alpha ax+b\beta y+c\gamma z$, $a\alpha^2+\beta^2b+c\lambda^2=1$, we get

$$x^2(a\alpha^2+c\lambda^2)a+2a\alpha b\beta xy+y^2(b\beta^2+c\gamma^2)b=0,$$

$$\frac{x}{y}=-\frac{a\alpha b\beta\pm\gamma\sqrt{(-abc)}}{a(1-b\beta^2)},\quad \frac{x}{z}=-\frac{a\alpha c\lambda\pm\beta\sqrt{(-abc)}}{a(1-c\gamma^2)},$$

$$\frac{y}{z}=-\frac{b\beta c\gamma\pm\alpha\sqrt{(-abc)}}{b(1-a\alpha^2)}.$$

Assuming a, b, c, α, β, γ to be real, then in order that $x:y:z$ may be real, $\sqrt{(-abc)}$ must be real. From $ax^2+by^2+cz^2=0$, it is clear that a, b, c can not all have the same sign and hence we must have one of the quantities a, b, c negative and the other two positive.

184. Proposed by J. A. CALDERHEAD, B.Sc., Professor of Mathematics, Curry University, Pittsburg, Pa.

If m rows, viz., the h_1th, h_2th, ..., h_mth, be transferred so as to become the 1st, 2nd, ..., mth, without altering the relative positions of the remaining rows, and that n columns, viz., the k_1th, k_2th, ..., k_nth, be similarly transformed the determinant thus obtained is the same as the original or differs from it only in sign according as $h_1+h_2+\dots+h_m-\frac{1}{2}m(m+1)+k_1+k_2+\dots+k_n-\frac{1}{2}n(n+1)$ is odd or even. [*Muir.*]

Solution by G. W. GREENWOOD, B. A. (Oxon), G. B. M. ZERR, A. M., Ph. D., and H. B. LEONARD, B. S.

In transferring the pth row (or column) to the qth row (or column) there are $p-q$ interchanges of adjacent rows (or columns) and therefore $p-q$ changes of sign. Hence, in the given example, there are

$$(h_1-1)+(h_2-2)+\dots+(h_m-m)+(k_1-1)+(k_2-2)+\dots+(k_n-n),$$
$$i.\ e.,\ h_1+h_2+\dots+h_m-\tfrac{1}{2}m(m+1)+k_1+k_2+\dots+k_n-\tfrac{1}{2}n(n+1)$$

changes of sign, and the determinant is unaltered in value, or differs only in sign, according as this value is *even* or *odd*; not odd or even as stated.

185. Proposed by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

Without introducing radicals, eliminate x and y from the equations (1) $ax^2+bx+c=0$, (2) $ay^2+by+d=0$, and (3) $ax^2y^2+bxy+e=0$.

I. Solution by H. F. MacNEISH, A. B., Instructor in Mathematics, University High School, Chicago, Ill., G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va., and J. E. SAUNDERS, Hackney, Ohio,

The eliminant of (1) and (3) is

$$\begin{vmatrix} a, & b, & c, & 0 \\ 0, & a, & b, & c \\ ay^2, & by, & e, & 0 \\ 0, & ay^2, & by, & e \end{vmatrix}=0$$